Physics Study Material

ATOMS

Updated for 2026 – 2027 Batch

Name: ______________________

Date: ______________________

Introduction

Coordinate Geometry is the branch of mathematics where algebra meets geometry – every point becomes a pair of numbers, and every geometric shape becomes an equation. The chapter **Straight Lines** is our first serious excursion into this world, after having studied the coordinate plane and distance/section formulae in Class 10 and the Coordinate Axes chapter earlier.

A straight line is the simplest possible curve – a set of points satisfying a *first-degree (linear) equation* in x and y. In this chapter, we develop a complete algebraic toolkit to describe, compare, and measure lines: their steepness (slope), the angle they make with each other, and their different standard equations depending on what information is given to us (a point and a slope, two points, an intercept, a perpendicular distance, and so on).

Big Idea of the Chapter

Every straight line in a plane can be represented by a linear equation $Ax+By+C=0$, and conversely, every linear equation in x, y represents a straight line. This equivalence between *geometry (the line)* and *algebra (the equation)* is the foundation of the entire chapter – and indeed, of coordinate geometry itself.

What Will Students Learn in This Chapter?

- Slope (gradient) of a line and its physical meaning
- Slope using two given points
- Conditions for two lines to be parallel or perpendicular
- Angle between two lines
- Collinearity of three points using slope
- Various forms of the equation of a line:
  - Point-slope form
  - Two-point form
  - Slope-intercept form
  - Intercept form
  - Normal (perpendicular) form
- General equation of a line $Ax + By + C = 0$
- Converting general form into the other standard forms
- Distance of a point from a line
- Distance between two parallel lines
- Locus and equation of locus (introductory idea)

Why Is This Chapter Useful?

Within Mathematics:
Straight Lines gives you the algebraic language to describe motion, boundaries, constraints and relationships that are linear in nature. Nearly every later topic in coordinate geometry builds directly on the definitions introduced here (slope, angle between lines, distance formulae).

Beyond Mathematics:
Linear relationships model countless real situations: cost vs. quantity, speed vs. time (uniform motion), depreciation of assets, and simple supply–demand relationships in economics – anywhere one quantity changes at a *constant rate* with respect to another.

Practical Applications

- **Architecture & construction:** laying out walls, roads, and roof slopes using gradients
- **Navigation:** plotting straight-line courses and bearings
- **Engineering & design:** structural supports, ramps, staircases (slope/gradient calculations)
- **Computer graphics:** drawing lines and edges of shapes on screen
- **Economics:** linear cost, revenue, and break-even analysis
- **Physics:** distance–time and velocity–time graphs (uniform motion), where slope represents speed or acceleration

Connection to Class 12 Mathematics

How This Chapter Builds Into Class 12

- **Conic Sections (Class 11, later chapter):** circles, parabolas, ellipses and hyperbolas are all studied using the same coordinate-geometry approach; tangents and normals to these curves are themselves *straight lines* whose equations you find using the exact methods from this chapter.
- **Application of Derivatives (Class 12):** the *equation of the tangent and normal* to a curve at a point uses the point-slope form of a line – learnt right here.
- **Three-Dimensional Geometry (Class 12):** direction cosines, direction ratios, and the equation of a line in 3D are direct extensions of slope and the two-point form from this 2D chapter.

- **Linear Programming (Class 12):** plotting constraint inequalities as straight lines and identifying feasible regions relies entirely on the equation-of-a-line skills built here.
- **Calculus (Class 12):** the idea that the derivative at a point gives the *slope of the tangent* connects straight-line slope to the rate of change – one of the most important ideas in all of Class 12 Mathematics.

Objectives of the Chapter

Learning Objectives

By the end of this chapter, a student should be able to:

1. Define and compute the slope of a line through two given points.
2. State and apply conditions for two lines to be parallel or perpendicular using their slopes.
3. Find the angle between two given lines.
4. Check whether three given points are collinear using slope.
5. Derive and use the various standard forms of the equation of a line (point-slope, two-point, slope-intercept, intercept, and normal forms).
6. Convert the general linear equation $Ax+By+C=0$ into any of the standard forms, and identify its slope and intercepts.
7. Calculate the perpendicular distance of a point from a line.
8. Calculate the distance between two parallel lines.
9. Apply the equation of a straight line to solve geometric and real-life problems involving lines.

Distance Between Two Points

Before we can study lines, we must be able to measure the distance between any two points in the coordinate plane. This single formula is the foundation for almost every result in coordinate geometry – including the slope formula, section formula, and later, distance of a point from a line.

Derivation

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the plane. Draw a line segment PQ, and construct a right-angled triangle by drawing a horizontal line from P and a vertical line from Q, meeting at point $R(x_2, y_1)$.

In right triangle PRQ:

$$PR = |x_2 - x_1| \quad \text{(horizontal leg)}$$
$$RQ = |y_2 - y_1| \quad \text{(vertical leg)}$$

By the **Pythagoras Theorem**:

$$PQ^2 = PR^2 + RQ^2$$
$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\boxed{PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

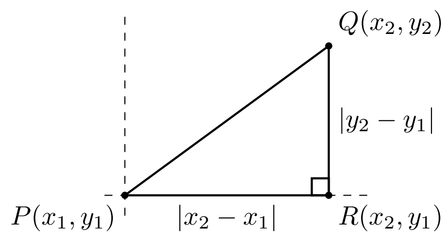


Figure 1

Key Formula: Distance Between Two Points

$$d = PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where $P(x_1, y_1)$ and $Q(x_2, y_2)$ are any two points in the plane.

Special Case: Distance From the Origin

If one of the two points is the origin $O(0, 0)$ and the other is $P(x, y)$, substituting $x_1 = y_1 = 0$ into the formula gives:

$$OP = \sqrt{x^2 + y^2}$$

This is simply the distance of a point from the origin, and is used very frequently as a quick check or shortcut in problems.

Quick Recall

- Distance is always **non-negative**.
- $PQ = QP$ – order of points does not matter.
- If $PQ = 0$, then P and Q are the same point.

Applications of the Distance Formula

The distance formula alone can be used to solve a surprising number of geometry problems, without needing any other tool.

- **Collinearity of three points:** A, B, C are collinear if $AB + BC = AC$ (one distance equals the sum of the other two).
- **Type of triangle:** compare all three side lengths – *scalene* (all different), *isosceles* (two equal), *equilateral* (all equal). Check $AB^2 + BC^2 = AC^2$ for a *right triangle.*
- **Type of quadrilateral:** compare all four sides and both diagonals to identify a square, rectangle, rhombus, or parallelogram.
- **Locus problems:** finding the path traced by a point that satisfies a given distance condition (e.g., equidistant from two fixed points).

Worked Examples

Example 1. Find the distance between $A(3, 2)$ and $B(-1, 5)$.
Solution:

$$AB = \sqrt{(-1 - 3)^2 + (5 - 2)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

Example 2. Show that the points $A(1, 3)$, $B(4, 7)$, $C(7, 3)$ form an isosceles triangle.
Solution:

$$AB = \sqrt{(4 - 1)^2 + (7 - 3)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$
$$BC = \sqrt{(7 - 4)^2 + (3 - 7)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$
$$AC = \sqrt{(7 - 1)^2 + (3 - 3)^2} = \sqrt{36} = 6$$

Since $AB = BC = 5 \neq AC$, triangle ABC is **isosceles**.

Common Mistakes to Avoid

- Forgetting to square *both* differences before adding – a very common slip under exam pressure.
- Mixing up $(x_2 - x_1)$ with $(y_2 - y_1)$ when substituting values.
- Leaving the answer as $\sqrt{\cdot}$ un-simplified when it simplifies to a whole number or simpler surd.
- Assuming $AB + BC = AC$ automatically without actually computing all three distances – collinearity must be checked numerically.

DPP - I

Daily Practice Problems

Straight Lines — Distance Formula & Section Formula

Instructions

Attempt all questions. Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and the section formula wherever needed. Show complete working. Answers are given at the end — do not check them until you have attempted the question.

1. Find the distance between the points:

(i) $A(2, -3)$ and $B(-6, 3)$

(ii) $C(-1, -1)$ and $D(8, 11)$

(iii) $P(-8, -3)$ and $Q(-2, -5)$

(iv) $R(a + b,\, a - b)$ and $S(a - b,\, a + b)$

2. If a point $P(x, y)$ is equidistant from the points $A(6, -1)$ and $B(2, 3)$, find the relation between x and y.

3. Find a point on the x-axis which is equidistant from the points $A(7, 6)$ and $B(-3, 4)$.

4. Find the distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$, when (i) AB is parallel to the x-axis (ii) AB is parallel to the y-axis.

5. A is a point on the x-axis with abscissa -8 and B is a point on the y-axis with ordinate 15. Find the distance AB.

6. Find a point on the y-axis which is equidistant from $A(-4, 3)$ and $B(5, 2)$.

7. Using the distance formula, show that the points $A(3, -2)$, $B(5, 2)$ and $C(8, 8)$ are collinear.

8. Show that the points $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$ are the vertices of an isosceles right-angled triangle.

9. Show that the points $A(1, 1)$, $B(-1, -1)$ and $C(-\sqrt{3}, \sqrt{3})$ are the vertices of an equilateral triangle each of whose sides is $2\sqrt{2}$ units.

10. Show that $A(3, 2)$, $B(0, 5)$, $C(-3, 2)$ and $D(0, -1)$ are the vertices of a square.

11. Show that the points $A(2, -1)$, $B(3, 4)$, $C(-2, 3)$ and $D(-3, -2)$ are the vertices of a rhombus.

12. If the points $A(-2, -1)$, $B(1, 0)$, $C(x, 3)$ and $D(1, y)$ are the vertices of a parallelogram, find the values of x and y.

13. Find the area of $\triangle ABC$ whose vertices are $A(-3, -5)$, $B(5, 2)$ and $C(-9, -3)$.

14. Show that the points $A(-5, 1)$, $B(5, 5)$ and $C(10, 7)$ are collinear.

15. Find the value of k for which the points $A(-2, 3)$, $B(1, 2)$ and $C(k, 0)$ are collinear.

16. Find the area of the quadrilateral whose vertices are $A(-4, 5)$, $B(0, 7)$, $C(5, -5)$ and $D(-4, -2)$.

17. Find the area of $\triangle ABC$, the midpoints of whose sides AB, BC and CA are $D(3, -1)$, $E(5, 3)$ and $F(1, -3)$ respectively.

18. Find the coordinates of the point which divides the join of $A(-5, 11)$ and $B(4, -7)$ in the ratio $2 : 7$.

19. Find the ratio in which the x-axis cuts the join of the points $A(4, 5)$ and $B(-10, -2)$. Also, find the point of intersection.

20. In what ratio is the line segment joining the points $A(-4, 2)$ and $B(8, 3)$ divided by the y-axis? Also, find the point of intersection.

Ph No: 9731385870
Landline: 080-26394711

SPARK SCIENCE FOUNDATION
Padmanabhanagar (Kidney Foundation Road)
Bengaluru - 560070

Answers – DPP

1. (i) 10 units (ii) 15 units
(iii) $2\sqrt{10}$ units (iv) $2\sqrt{2}\,b$ units

2. $x - y = 3$

3. $P(3, 0)$

4. (i) $|x_2 - x_1|$ (ii) $|y_2 - y_1|$

5. 17 units

6. $P(0, -2)$

7. Show $AB + BC = AC$

8. Show $AB = BC$ and $AB^2 + BC^2 = AC^2$

9. Show $AB = BC = CA = 2\sqrt{2}$

10. Show $AB = BC = CD = DA$, diag. AC = diag. BD

11. Show all four sides equal, diagonals unequal

12. $x = 4,\ y = 2$

13. 29 sq units

14. Show area of $\triangle ABC = 0$

15. $k = 7$

16. 60.5 sq units

17. 8 sq units

18. $P(-3, 7)$

19. Ratio 5 : 2; $P(-6, 0)$

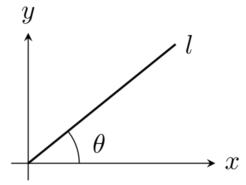
20. Ratio 1 : 2; $P\left(0, \dfrac{7}{3}\right)$

Straight Lines

Topic: Slope and Inclination of a Line

Inclination of a Line

Every non-horizontal, non-vertical line drawn in the plane makes some angle with the x-axis. This angle is what we call the **inclination** of the line, and it is the starting point for defining slope.



Definition. If a line l makes an angle θ with the *positive direction of the x-axis*, measured in the **anti-clockwise** direction, then θ is called the **inclination** of the line l.

By convention, the inclination of a line always lies in the range

$$0^\circ \le \theta < 180^\circ.$$

Special Inclinations

- Inclination of the x-axis (or any line parallel to it) is $\theta = 0^\circ$.
- Inclination of the y-axis (or any line parallel to it) is $\theta = 90^\circ$.

Slope (Gradient) of a Line

Definition. If θ is the inclination of a line l, then $\tan\theta$ is called the **slope** (or **gradient**) of the line, usually denoted by m.

Key Formula: Slope in Terms of Inclination

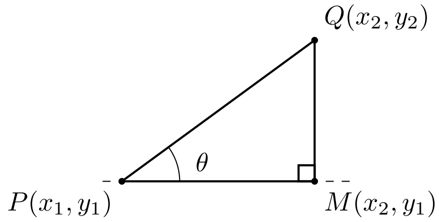
$$m = \tan\theta, \qquad \theta \neq 90^\circ$$

The slope is undefined when $\theta = 90^\circ$, i.e. for a line parallel to the y-axis, since $\tan 90^\circ$ is not defined.

Slope measures how steeply a line rises or falls: larger $|m|$ means a steeper line; positive m means the line rises left to right, negative m means it falls.

Slope When Two Points on the Line Are Known

In practice we are usually given two points on a line, not its angle of inclination. The following formula lets us compute the slope directly.



Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a non-vertical line l with inclination θ. Since the line is non-vertical, $x_1 \neq x_2$.
Drawing QR perpendicular to the x-axis and PM perpendicular to RQ, triangle PMQ is right-angled at M, and

$$\tan\theta = \frac{MQ}{PM} = \frac{y_2 - y_1}{x_2 - x_1}$$

(A similar argument, using $180^\circ - \theta$, gives the same formula when θ is obtuse.)

Key Formula: Slope Through Two Points

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2$$

where $P(x_1, y_1)$ and $Q(x_2, y_2)$ are any two points on the line.

Important Results on Slope

- Slope of a line parallel to the x-axis: $m = 0$.
- Slope of a line parallel to the y-axis: **undefined** (not defined).
- A line equally inclined to both axes has inclination 45° or 135°, so its slope is 1 or -1.
- The slope computed from any two points on a line is the same, regardless of which two points are chosen — slope is a property of the *whole line*, not of individual points.
- Three points A, B, C are collinear if and only if slope of AB = slope of BC.

Worked Examples

Example 1. Find the slope of the line joining $A(2, 3)$ and $B(-4, 1)$.

Solution:

$$m = \frac{1 - 3}{-4 - 2} = \frac{-2}{-6} = \frac{1}{3}$$

Example 2. Using slopes, show that $A(2, -1)$, $B(4, 3)$ and $C(6, 7)$ are collinear.

Solution:

$$m_{AB} = \frac{3 - (-1)}{4 - 2} = \frac{4}{2} = 2$$

$$m_{BC} = \frac{7 - 3}{6 - 4} = \frac{4}{2} = 2$$

Since $m_{AB} = m_{BC}$, points A, B, C are **collinear**.

Common Mistakes to Avoid

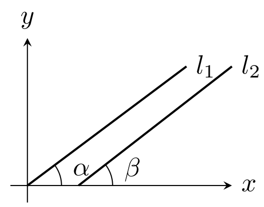
- Subtracting coordinates in the wrong order in numerator vs. denominator — e.g. using $y_2 - y_1$ over $x_1 - x_2$. Keep the *same order* in both.
- Saying the slope of a vertical line is 0 — it is actually **undefined**. (It is the slope of a *horizontal* line that is 0.)
- Confusing inclination (θ, an angle) with slope ($m = \tan\theta$, a number) — they are related but not the same thing.
- Forgetting that inclination is always taken between 0° and 180°, never reflex or negative.

Straight Lines

Topic: Slopes of Parallel and Perpendicular Lines

We already know that the slope of a line is $m = \tan\theta$, where θ is its inclination. We now use this to answer a very natural question: *what does slope tell us about the relationship between two lines?* In particular, we derive simple slope conditions that tell us instantly whether two lines are parallel or perpendicular — without drawing anything.

Condition for Two Lines to Be Parallel



Let l_1 and l_2 be two lines with inclinations α and β, and slopes $m_1 = \tan\alpha$ and $m_2 = \tan\beta$ respectively.

If $l_1 \parallel l_2$, the two lines never meet and rise at exactly the same steepness, so their inclinations must be equal:

$$\alpha = \beta$$

Taking tangent of both sides:

$$\tan\alpha = \tan\beta \quad \Longrightarrow \quad m_1 = m_2$$

Key Formula: Parallel Lines

$$l_1 \parallel l_2 \quad \Longleftrightarrow \quad m_1 = m_2$$

Two non-vertical lines are parallel if and only if their slopes are equal.

Condition for Two Lines to Be Perpendicular

Let l_1 and l_2 have inclinations α and β, and slopes $m_1 = \tan\alpha$, $m_2 = \tan\beta$.

If $l_1 \perp l_2$, then l_2 is inclined exactly $90°$ more than l_1:

$$\beta = \alpha + 90°$$

Taking tangent of both sides:

$$\tan\beta = \tan(\alpha + 90°) = -\cot\alpha = -\frac{1}{\tan\alpha}$$

$$\Longrightarrow \quad \tan\alpha \cdot \tan\beta = -1 \quad \Longrightarrow \quad m_1 m_2 = -1$$

Key Formula: Perpendicular Lines

$$l_1 \perp l_2 \quad \Longleftrightarrow \quad m_1 \cdot m_2 = -1$$

i.e. the slope of one line is the **negative reciprocal** of the other: $m_2 = -\dfrac{1}{m_1}$.

Special Case — Horizontal and Vertical Lines

The rule $m_1 m_2 = -1$ assumes *both* slopes exist. A line parallel to the x-axis ($m = 0$) and a line parallel to the y-axis (slope undefined) are always perpendicular to each other, but this cannot be checked using $m_1 m_2 = -1$ since one slope is undefined — it must be recognised directly from the diagram.

Where These Conditions Are Used

- Proving that two given lines are parallel or perpendicular.
- Identifying a quadrilateral's type: e.g. opposite sides parallel (parallelogram), adjacent sides perpendicular (rectangle/square).
- Finding the equation of a line parallel or perpendicular to a given line (studied in the next topic).
- Checking whether a triangle is right-angled, by testing if any two sides have slopes multiplying to -1.

Worked Examples

Example 1. Show that the line through $A(1,1)$ and $B(3,5)$ is parallel to the line through $C(-1,-1)$ and $D(1,3)$.
Solution:

$$m_{AB} = \frac{5-1}{3-1} = \frac{4}{2} = 2$$

$$m_{CD} = \frac{3-(-1)}{1-(-1)} = \frac{4}{2} = 2$$

Since $m_{AB} = m_{CD}$, $AB \parallel CD$.

Example 2. Show that the line through $A(1,1)$ and $B(3,5)$ is perpendicular to the line through $C(2,6)$ and $D(6,4)$.
Solution:

$$m_{AB} = \frac{5-1}{3-1} = 2$$

$$m_{CD} = \frac{4-6}{6-2} = \frac{-2}{4} = -\frac{1}{2}$$

Since $m_{AB} \cdot m_{CD} = 2 \times \left(-\frac{1}{2}\right) = -1$, $AB \perp CD$.

Common Mistakes to Avoid

- Writing the perpendicular condition as $m_1 = -m_2$ instead of $m_1 m_2 = -1$ — these are different unless $|m_1| = 1$.
- Forgetting to *invert* as well as change sign when finding a perpendicular slope: the perpendicular slope is $-\frac{1}{m}$, not just $-m$.
- Assuming two lines are perpendicular just because they "look" perpendicular in a rough sketch — always verify using $m_1 m_2 = -1$.
- Trying to apply $m_1 m_2 = -1$ when one of the lines is vertical (undefined slope) — use the special case instead.

DPP - II

Daily Practice Problems

Based on slope and inclination

1. Find the slope of a line whose inclination is
   (i) $30°$
   (ii) $120°$
   (iii) $135°$
   (iv) $90°$
2. Find the inclination of a line whose slope is
   (i) $\sqrt{3}$
   (ii) $\frac{1}{\sqrt{3}}$
   (iii) 1
   (iv) -1
   (v) $-\sqrt{3}$
3. Find the slope of a line which passes through the points
   (i) $(0,0)$ and $(4,-2)$
   (ii) $(0,-3)$ and $(2,1)$
   (iii) $(2,5)$ and $(-4,-4)$
   (iv) $(-2,3)$ and $(4,-6)$
4. If the slope of the line joining the points $A(x,2)$ and $B(6,-8)$ is $-\frac{5}{4}$, find the value of x.
5. Show that the line through the points $(5,6)$ and $(2,3)$ is parallel to the line through the points $(9,-2)$ and $(6,-5)$.
6. Find the value of x so that the line through $(3,x)$ and $(2,7)$ is parallel to the line through $(-1,4)$ and $(0,6)$.
7. Show that the line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line through the points $(3,-3)$ and $(5,-9)$.
8. If $A(2,-5)$, $B(-2,5)$, $C(x,3)$ and $D(1,1)$ be four points such that AB and CD are perpendicular to each other, find the value of x.
9. Without using Pythagoras' theorem, show that the points $A(1,2)$, $B(4,5)$ and $C(6,3)$ are the vertices of a right-angled triangle.
10. Using slopes, show that the points $A(6,-1)$, $B(5,0)$ and $C(2,3)$ are collinear.
11. Using slopes, find the value of x for which the points $A(5,1)$, $B(1,-1)$ and $C(x,4)$ are collinear.

12. Using slopes, show that the points $A(-4,-1)$, $B(-2,-4)$, $C(4,0)$ and $D(2,3)$ taken in order, are the vertices of a rectangle.

13. If the three points $A(h,k)$, $B(x_1,y_1)$ and $C(x_2,y_2)$ lie on a line, show that
$$(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1).$$

14. If the points $A(a,0)$, $B(0,b)$ and $P(x,y)$ are collinear, using slopes, prove that
$$\frac{x}{a}+\frac{y}{b}=1.$$

15. A line passes through the points $A(4,-6)$ and $B(-2,-5)$. Show that the line AB makes an obtuse angle with the x-axis.

16. The vertices of a quadrilateral are $A(-4,2)$, $B(2,6)$, $C(8,5)$ and $D(9,-7)$. Using slopes, show that the midpoints of the sides of the quadrilateral $ABCD$ form a parallelogram.

17. Find the slope of the line which makes an angle of $30°$ with the positive direction of the y-axis, measured anticlockwise.

18. Find the angle between the lines whose slopes are $\sqrt{3}$ and $\dfrac{1}{\sqrt{3}}$.

19. Find the angle between the lines whose slopes are $(2-\sqrt{3})$ and $(2+\sqrt{3})$.

20. If $A(1,2)$, $B(-3,2)$ and $C(3,-2)$ be the vertices of a $\triangle ABC$, show that
    (i) $\tan A = 2$
    (ii) $\tan B = \dfrac{2}{3}$
    (iii) $\tan C = \dfrac{4}{7}$

21. If θ is the angle between the lines joining the points $A(0,0)$ and $B(2,3)$, and the points $C(2,-2)$ and $D(3,5)$, show that
$$\tan\theta = \frac{11}{23}.$$

22. If θ is the angle between the diagonals of a parallelogram $ABCD$ whose vertices are $A(0,2)$, $B(2,-1)$, $C(4,0)$ and $D(2,3)$, show that
$$\tan\theta = 2.$$

23. Show that the points $A(0,6)$, $B(2,1)$ and $C(7,3)$ are three corners of a square $ABCD$. Find
    (i) the slope of the diagonal BD,
    (ii) the coordinates of the fourth vertex D.

24. $A(1,1)$, $B(7,3)$ and $C(3,6)$ are the vertices of a $\triangle ABC$. If D is the midpoint of BC and $AL \perp BC$, find the slopes of
    (i) AD,
    (ii) AL.

Answers – Exercise 20B

1. (i) $\dfrac{1}{\sqrt{3}}$ (ii) $-\sqrt{3}$ (iii) -1 (iv) Not defined

2. (i) $60°$ (ii) $30°$ (iii) $45°$ (iv) $135°$ (v) $120°$

3. (i) $-\dfrac{1}{2}$ (ii) 2 (iii) $\dfrac{3}{2}$ (iv) $-\dfrac{3}{2}$

4. $x=-2$

5. Show that the two lines are parallel.

6. $x=9$

7. Show that the two lines are perpendicular.

8. $x=6$

9. Show that $\triangle ABC$ is right-angled.

10. Show that the points are collinear.

11. $x=11$

12. Show that $ABCD$ is a rectangle.

13. Show that
$$(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1).$$

14. Show that
$$\frac{x}{a}+\frac{y}{b}=1.$$

15. Show that AB makes an obtuse angle with the x-axis.

16. Show that the midpoints form a parallelogram.

17. $-\sqrt{3}$

18. $30°$

19. $60°$

21. (i) $\tan A = 2$ (ii) $\tan B = \dfrac{2}{3}$ (iii) $\tan C = \dfrac{4}{7}$

20. Show that
$$\tan\theta = \frac{11}{23}.$$

21. Show that
$$\tan\theta = 2.$$

22. (i) $\dfrac{7}{3}$ (ii) $(5,8)$

23. (i) $\dfrac{7}{8}$ (ii) $\dfrac{4}{3}$

Various Forms of the Equation of a Line

A straight line can be represented by different equations depending on the information given about the line. The important forms are:

Form	Given information	Equation
Horizontal line	Distance from x-axis	$y = a$
Vertical line	Distance from y-axis	$x = a$
Point–slope	Point and slope	$y - y_1 = m(x - x_1)$
Two–point	Two points	$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
Slope–intercept	Slope and y-intercept	$y = mx + c$
Intercept	x- and y-intercepts	$\dfrac{x}{a} + \dfrac{y}{b} = 1$
Normal	Perpendicular distance and angle	$x\cos\omega + y\sin\omega = p$

1. Horizontal and Vertical Lines

Horizontal Line
A horizontal line is parallel to the x-axis.
Hence its slope is
$$m = 0.$$
If it passes through (x_1, y_1), then
$$\boxed{y = y_1}.$$
Thus, a horizontal line has the general form
$$\boxed{y = a},$$
where a is a constant.

Vertical Line
A vertical line is parallel to the y-axis.
Its slope is **not defined**.
If it passes through (x_1, y_1), then
$$\boxed{x = x_1}.$$
Thus, a vertical line has the general form
$$\boxed{x = a},$$
where a is a constant.

Remember

$\boxed{\text{Horizontal line: } y = a, \quad m = 0}$ $\boxed{\text{Vertical line: } x = a, \quad m \text{ is not defined}}$

2. Point–Slope Form

Suppose a line has slope m and passes through the fixed point $P(x_1, y_1)$.
Let $Q(x, y)$ be any point on the line.
Since the slope of PQ is m,
$$m = \frac{y - y_1}{x - x_1}$$
$$\therefore \quad \boxed{y - y_1 = m(x - x_1)}.$$
This is called the **point–slope form** of the equation of a line.

Point–Slope Form

$$\boxed{y - y_1 = m(x - x_1)}$$

Use when: one point (x_1, y_1) and the slope m are known.

Special case: If the line passes through (x_1, y_1) and is horizontal, then $m = 0$:
$$y - y_1 = 0 \implies y = y_1.$$

3. Two–Point Form

Suppose a line passes through two distinct points

$$P_1(x_1, y_1), \qquad P_2(x_2, y_2),$$

where $x_1 \neq x_2$.

Let $P(x, y)$ be any point on the line.

Since P_1, P, P_2 are collinear,

$$\text{slope of } P_1P = \text{slope of } P_1P_2.$$

Hence,

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Therefore,

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}.$$

Equivalently,

$$\boxed{\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}}.$$

Two–Point Form

$$\boxed{\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}}$$

or

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

Use when: two points on the line are known.

Note: If $x_1 = x_2$, the line is vertical and its equation is

$$\boxed{x = x_1}.$$

4. Slope–Intercept Form

Suppose a line has slope m and cuts the y-axis at the point $(0, c)$.

Here c is called the **y-intercept**.

Using the point–slope form,

$$y - c = m(x - 0)$$

$$\therefore \quad \boxed{y = mx + c}.$$

This is called the **slope–intercept form**.

Slope–Intercept Form

$$\boxed{y = mx + c}$$

where

$$\boxed{m = \text{slope}}, \qquad \boxed{c = y\text{-intercept}}.$$

If the line cuts the y-axis at $(0, c)$, then

$$\boxed{y\text{-intercept} = c}.$$

Special cases:

$$c = 0 \implies y = mx$$

gives a line passing through the origin, while

$$m = 0 \implies y = c$$

gives a horizontal line.

5. Intercept Form

Suppose a line cuts the x-axis at $(a, 0)$ and the y-axis at $(0, b)$.

Thus,

$$x\text{-intercept} = a, \qquad y\text{-intercept} = b.$$

The slope of the line is

$$m = \frac{b-0}{0-a} = -\frac{b}{a}.$$

Using the point–slope form through $(a, 0)$,

$$y = -\frac{b}{a}(x-a)$$

$$ay = -bx + ab$$

$$bx + ay = ab.$$

Dividing by ab,

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}.$$

This is called the **intercept form**.

Intercept Form

If a line makes intercepts a and b on the x- and y-axes, respectively, then

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}.$$

$$\boxed{x\text{-intercept} = a, \qquad y\text{-intercept} = b}$$

Important: The intercepts a and b are *signed* quantities. Hence they may be positive or negative.

Special cases: If the line passes through the origin, the intercept form is not applicable because one or both intercepts may be zero.

7. Conversion Between Forms

Point–Slope → Slope–Intercept

$$y - y_1 = m(x - x_1)$$

$$\implies y - y_1 = mx - mx_1$$

$$\boxed{y = mx + (y_1 - mx_1)}.$$

Hence the y-intercept is

$$\boxed{c = y_1 - mx_1}.$$

Slope–Intercept → Point–Slope

Given

$$y = mx + c,$$

the line passes through $(0, c)$.

Hence,

$$\boxed{y - c = m(x - 0)}.$$

8. Important Special Cases

Condition	Equation
Line parallel to x-axis	$y = a$
Line parallel to y-axis	$x = a$
Line passing through origin with slope m	$y = mx$
Line passing through (x_1, y_1) with slope m	$y - y_1 = m(x - x_1)$
Line through $(x_1, y_1), (x_2, y_2)$	$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
Line with slope m and y-intercept c	$y = mx + c$
Line with intercepts a, b	$\frac{x}{a} + \frac{y}{b} = 1$

9. How to Choose the Correct Form

Quick Selection Rule

- **One point + slope** $\implies$ use point–slope form.
- **Two points** $\implies$ use two–point form.
- **Slope + y-intercept** $\implies$ use slope–intercept form.
- **x- and y-intercepts** $\implies$ use intercept form.
- **Perpendicular distance from origin + angle of normal** $\implies$ use normal form.
- **Parallel to x-axis** $\implies$ $y = a$.
- **Parallel to y-axis** $\implies$ $x = a$.

11. Key Results at a Glance

MASTER FORMULA BOX

$$\boxed{y = a}$$

Horizontal line.

$$\boxed{x = a}$$

Vertical line.

$$\boxed{y - y_1 = m(x - x_1)}$$

Point–slope form.

$$\boxed{\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}}$$

Two–point form.

$$\boxed{y = mx + c}$$

Slope–intercept form.

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

Intercept form.

Angle Between Two Lines

Let L_1 and L_2 be two non-vertical lines having slopes m_1 and m_2, respectively. Let their inclinations with the positive direction of the x-axis be α_1 and α_2.

Then

$$m_1 = \tan \alpha_1, \qquad m_2 = \tan \alpha_2.$$

If the acute angle between the two lines is θ, then

$$\theta = |\alpha_2 - \alpha_1|.$$

Therefore,

$$\tan \theta = |\tan(\alpha_2 - \alpha_1)|$$

$$\therefore \quad \boxed{\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|}, \qquad 1 + m_1 m_2 \neq 0.$$

Angle Between Two Lines

If the slopes of two lines are m_1 and m_2, then the **acute angle** θ between them is given by

$$\boxed{\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|}$$

provided

$$\boxed{1 + m_1 m_2 \neq 0}.$$

1. Why the Formula Works

Since

$$m_1 = \tan\alpha_1, \qquad m_2 = \tan\alpha_2,$$

and

$$\theta = |\alpha_2 - \alpha_1|,$$

we have

$$\tan\theta = \left|\frac{\tan\alpha_2 - \tan\alpha_1}{1 + \tan\alpha_1 \tan\alpha_2}\right|$$

$$\therefore \quad \boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|.}$$

The modulus is used because the formula gives the **acute angle** between the two lines.

Important

Two intersecting lines form two angles:

$$\boxed{\theta \quad \text{and} \quad 180^\circ - \theta}.$$

If θ is the acute angle, then

$$\boxed{\text{obtuse angle} = 180^\circ - \theta}.$$

2. Finding the Angle Step-by-Step

Procedure

1. Find the slopes m_1 and m_2 of the two lines.
2. Use
$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|.$$
3. Find the acute angle θ.
4. If the obtuse angle is required, use
$$180^\circ - \theta.$$

3. Special Cases

(i) Parallel Lines

For parallel lines,

$$\theta = 0^\circ.$$

Hence,

$$\tan\theta = 0$$

$$\therefore \quad m_2 - m_1 = 0$$

$$\boxed{m_1 = m_2}.$$

Thus, two non-vertical lines are parallel iff

$$\boxed{m_1 = m_2}.$$

(ii) Perpendicular Lines

For perpendicular lines,

$$\theta = 90^\circ.$$

Thus the denominator in the angle formula must be zero:

$$1 + m_1 m_2 = 0.$$

Hence,

$$\boxed{m_1 m_2 = -1}.$$

Thus, two non-vertical lines are perpendicular iff

$$\boxed{m_1 m_2 = -1}.$$

Conditions to Remember

$$\boxed{\text{Parallel} \iff m_1 = m_2} \qquad \boxed{\text{Perpendicular} \iff m_1 m_2 = -1}$$

4. Angle Between Two Lines in Terms of Their Inclinations

If the inclinations of the two lines are α_1 and α_2, then the angles between them are

$$\boxed{|\alpha_2 - \alpha_1|}$$

and

$$\boxed{180^\circ - |\alpha_2 - \alpha_1|}.$$

Hence, the acute angle is

$$\boxed{\theta = |\alpha_2 - \alpha_1|}$$

when $|\alpha_2 - \alpha_1| \leq 90^\circ$.

Otherwise, the acute angle is

$$\boxed{180^\circ - |\alpha_2 - \alpha_1|}.$$

5. When the Equations of the Lines are Given

Suppose the equations of two lines are

$$L_1 : A_1x + B_1y + C_1 = 0$$

and

$$L_2 : A_2x + B_2y + C_2 = 0.$$

Their slopes are

$$\boxed{m_1 = -\frac{A_1}{B_1}}, \qquad \boxed{m_2 = -\frac{A_2}{B_2}},$$

provided $B_1, B_2 \neq 0$.

Using

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|,$$

we get

$$\tan\theta = \left|\frac{-\frac{A_2}{B_2} + \frac{A_1}{B_1}}{1 + \frac{A_1A_2}{B_1B_2}}\right|$$

$$= \left|\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right|.$$

Therefore,

Angle Between Lines in General Form

For

$$A_1x + B_1y + C_1 = 0$$

and

$$A_2x + B_2y + C_2 = 0,$$

the acute angle θ between them is given by

$$\boxed{\tan\theta = \left|\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right|}$$

provided

$$A_1A_2 + B_1B_2 \neq 0.$$

6. Conditions Using General Equations

For

$$A_1x + B_1y + C_1 = 0, \qquad A_2x + B_2y + C_2 = 0,$$

Parallel Lines

$$\tan\theta = 0$$

$$\therefore \quad A_1B_2 - A_2B_1 = 0.$$

Hence,

$$\boxed{A_1B_2 = A_2B_1}.$$

Perpendicular Lines

$$\theta = 90^\circ$$

$$\therefore \quad A_1A_2 + B_1B_2 = 0.$$

Hence,

$$\boxed{A_1A_2 + B_1B_2 = 0}.$$

8. Finding the Slope of a Line Making a Given Angle with Another Line

Suppose a line of slope m_2 makes an angle θ with a line of slope m_1.

Then

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1m_2}\right|.$$

Ignoring the modulus initially,

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1m_2}.$$

Let

$$t = \tan\theta.$$

Then

$$t(1 + m_1m_2) = m_2 - m_1$$

$$t + tm_1m_2 = m_2 - m_1$$

$$m_2(tm_1 - 1) = -m_1 - t$$

$$\boxed{m_2 = \frac{m_1 + t}{1 - m_1t}}.$$

Considering the other possible angle,

$$\boxed{m_2 = \frac{m_1 - \tan\theta}{1 + m_1\tan\theta}}.$$

Therefore, the two possible slopes of lines making an angle θ with a line of slope m are

$$\boxed{m' = \frac{m + \tan\theta}{1 - m\tan\theta}}$$

or

$$\boxed{m' = \frac{m - \tan\theta}{1 + m\tan\theta}}.$$

9. Quick Revision

MASTER FORMULA BOX

Slopes m_1, m_2:

$$\boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|}$$

Parallel:

$$\boxed{m_1 = m_2}$$

Perpendicular:

$$\boxed{m_1 m_2 = -1}$$

General equations:

$$A_1x + B_1y + C_1 = 0$$

$$A_2x + B_2y + C_2 = 0$$

$$\boxed{\tan\theta = \left|\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right|}$$

Parallel:

$$\boxed{A_1B_2 = A_2B_1}$$

Perpendicular:

$$\boxed{A_1A_2 + B_1B_2 = 0}$$

10. Common Mistakes

Common Mistakes to Avoid

1. Do not forget the modulus when the **acute angle** is required:

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|.$$

2. Do not use $m_1 m_2 = -1$ for parallel lines. For parallel lines,

$$m_1 = m_2.$$

3. If $m_1 m_2 = -1$, the lines are perpendicular and

$$\boxed{\theta = 90^\circ}.$$

4. If the obtuse angle is required after finding the acute angle θ, use

$$\boxed{180^\circ - \theta}.$$

5. For a line $Ax + By + C = 0$, its slope is

$$\boxed{-\frac{A}{B}},$$

not $\dfrac{A}{B}$.